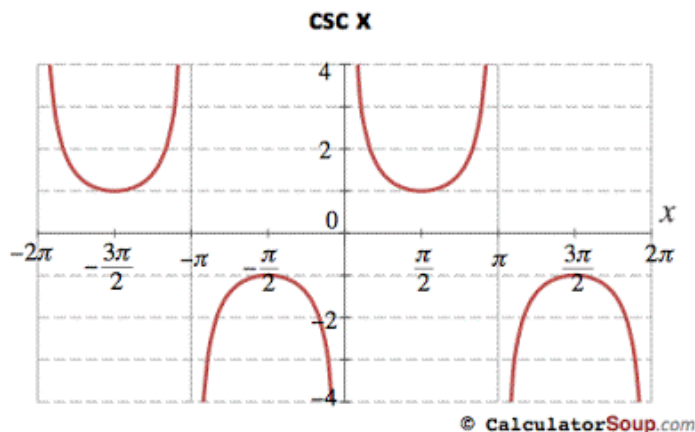# Implicit differentiation

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Problems

Problem 1. Find the formula for the derivative of $f(x) = \operatorname{arccsc}(x)$. (**Warning:** the restricted domain will consist of two intervals.)

CSC X

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Solution: The restricted domain is $(-\frac{\pi}{2}, \frac{\pi}{2})$. The derivative of $\csc(x)$ is $-\frac{\cos x}{\sin^2 x}$. We apply the Derivative of Inverse function theorem, get

$$(\operatorname{arccsc}(x))' = \frac{1}{\csc'(\operatorname{arccsc} x)} = \frac{-\sin^2(\operatorname{arccsc} x)}{\cos(\operatorname{arccsc} x)}$$

Now, $\operatorname{arccsc} x$ is the angle (call it α) such that $\csc(\alpha) = x$. So, $\frac{1}{\sin \alpha} = x$, and hence $\sin(\alpha) = \frac{1}{x}$.

Problem 2. Find the equation for the slope of the tangent line to the curve $y^4 = y^2 - x^2$ at a point (x_0, y_0). What are the points where the slope of the tangent line is 0?

Problem 3. Harry Potter casts a spell to make his aunt Marjorie fill in with air and fly up to the sky at a constant speed 2 feet/sec. At first, everyone just stares at aunt Marjorie rising: members of Dursley family are in shock, Harry is silently happy. As soon as aunt Marge reaches 50 feet above the ground, Harry Potter feels he should run away, so he starts running at a constant speed 16 feet/sec. How fast is the distance between Harry and aunt Marge is growing exactly 5 seconds after Harry started to run?

Problem 4. Pinocchio is running for President of his High school class. He is giving a speech promising that, if elected, he will make the minimum grade a student can get to be A-, free snacks at the school diner, and many other nice things. Lies constitute around 50% of the total number of words Pinocchio is saying. With every word of lie, Pinocchio's nose (which is cylindrical of initial diameter 1 inch and length 3 inches) is growing in length by 0.5 inches and by 0.05 inches in width. How fast is the volume of his nose growing after 10 minutes of his speech if you know that Pinocchio says about 150 words per minute.